10 Nature's Steam Engine

C'est à la chaleur que doivent être attribués les grands mouvements qui frappent nos regards sur la terre; c'est à elle que sont dues les agitations de l'atmosphère, l'ascension des nuages, la chute des pluies et des autres météores, les courants d'eau qui sillonnent la surface du globe et dont l'homme est parvenu à employer pour son usage une faible partie; enfin les tremblements de terre, les éruptions volcanique reconnaissent aussi pour cause le chaleur.

—Nicolas Léonard Sadi Carnot[6]

Thus the great nineteenth-century French scientist Carnot attributed to the flow of heat the grand motions of the atmosphere, the flow of rivers, and the forces inside the earth that drive earthquakes and volcanoes. Physicists such as Carnot were motivated, in part, by a desire to understand the theory underlying the operation of steam engines, and they were particularly concerned to determine how efficient such engines can be made. Carnot showed that for any engine to convert heat energy into mechanical energy, heat has to flow from a high-temperature reservoir to a low-temperature reservoir, and he suggested that the fraction of heat energy that can be converted is proportional to the difference between the temperatures of the two reservoirs.

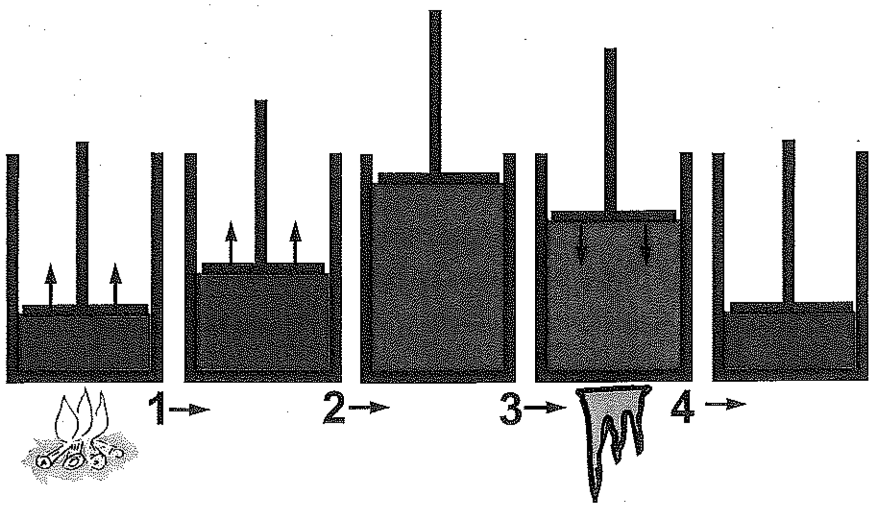
Carnot's other great contribution to thermodynamics was his demonstration that a particular kind of heat engine maximizes the conversion of heat energy to mechanical energy. This ideal heat engine works on a particular thermodynamic cycle in which a working substance is transformed in four steps, ending up in the same state that it began. He illustrated this cycle by considering a gas in a cylinder contained by a piston (Figure 10.1):

Step 1: The gas is heated and at the same time, the piston rises, reducing the pressure on the gas. When pressure falls, the gas cools, but in this case, just enough heat is added to keep the temperature of the gas constant. This is called *isothermal expansion.*

Step 2: The heating is turned off, but the piston continues to rise, further reducing the pressure on the gas. As the pressure is reduced, the gas cools. This is called *adiabatic expansion.*

[6]"It is to heat we should attribute the great movements that appear to us on earth; to it are due the agitations of the atmosphere, the rising of clouds, the fall of rain and other meteors, the water currents that furrow the surface of the globe and of which man has managed to turn a small part to his use; and finally, earthquakes and volcanic eruptions are also caused by heat." *Reflexions sur la Puissance Motrice du Feu,* Paris, 1824.

Figure 10.1: The four steps of the classical Carnot energy cycle: isothermal expansion; adiabatic expansion; isothermal compression; and adiabatic compression.



("Adiabatic" means "without addition of heat.")

Step 3: In the reverse of step 1, the gas is cooled and at the same time, the piston falls, increasing the pressure on the gas at just such a rate as to keep its temperature constant. This is called *isothermal compression*, and the gas loses heat during this step.

Step 4: In this last step, the cold source is removed, and the gas is compressed until it warms to the temperature it had at the start of the cycle. This is called *adiabatic compression*.

Carnot showed that this cycle, which now bears his name, does work on whatever is attached to the piston. So, for example, if the piston is connected to a crankshaft, some of the heat energy (supplied by the fire in this example) is used to accelerate the shaft or, if there is enough friction somewhere in the system, to keep the shaft turning against friction. As noted earlier, he also showed that the fraction of energy supplied by the heat source that is ultimately used to do this work is proportional to the difference between the temperatures of the substance (gas, in this case) in steps 1 and 3. The greater this temperature difference, the more work can be done for the same input of heat energy. This relation can be expressed mathematically. Let W be the mechanical work that can be done by the engine, and Q the rate of heat input. Also let T_{hot} be the absolute temperature[7] of the substance at the beginning of the cycle, and let T_{cold} be the temperature at the end of step 2. Then

$$W=Q\left(\frac{T_{hot}-T_{cold}}{T_{hot}}\right). \tag{10.1}$$

An automobile engine is an example of a heat engine. Heat is put into the engine when a spark plug ignites the mixture of gasoline and air inside each cylinder. This happens when the piston is nearly at the end of the cylinder, and the mixture is at maximum compression and maximum temperature. (In a diesel engine, the gases get so hot from compression that they ignite

[7] The absolute temperature is measured in degrees Kelvin, where 0 Kelvin is "absolute zero." This is the coldest temperature any substance can attain, at which all molecular motions cease. 0 Kelvin is equal to -273 C or -459 F.

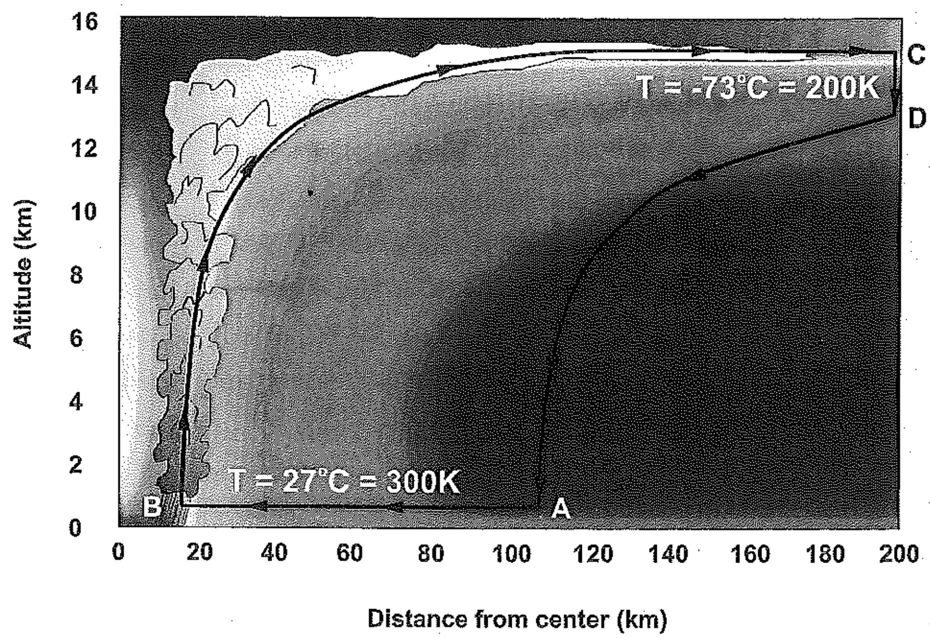


Figure 10.2: The energy cycle of a mature hurricane. Air spirals inward close to the sea surface, between points A and B, acquiring heat from the ocean by evaporation of sea-water. Air then ascends in the eyewall, from B to C, without acquiring or losing heat other than that produced when water vapor condenses. Between C and D, the air loses the heat it originally acquired from the ocean. Finally, between D and A, the air returns to its starting point. In a real hurricane, the energy cycle is open because hurricanes continuously exchange air with their environment (see text). The colors show a measure of the air's heat content, with warm colors corresponding to high heat content.

without a spark plug.) After the miniature explosion takes place, the gases in the cylinder expand and cool, and the piston is forced rapidly downward. Heat is then taken away from the cylinder by the circulation of cool water (or air, if it's an old VW Beetle) through the engine and ultimately exchanged to the atmosphere through the car's radiator.

The auto engine departs in several respects from the ideal heat engine first described by Carnot. But it turns out that a hurricane is an almost perfect example of a Carnot heat engine.

In a hurricane, the working substance is not just air, but a mixture of moist air, water droplets, and ice crystals. (We could choose dry air as the working substance, but then we would have to take into account the enormous conversion of energy that takes place when water vapor condenses into liquid water drops or ice crystals. By choosing moist air and condensed water as the working fluid, we can mostly avoid dealing with phase changes of water.) The heat cycle of the hurricane is shown in Figure 10.2.

This diagram shows a cross section through an idealized hurricane. The central axis of the storm is on the left side of the diagram. The wind speeds of the storm are assumed not to be changing in time, and we neglect any variations in the properties of the air as one moves around the axis of the storm. Let's follow a sample of air that begins at point A near the sea surface, a hundred kilometers or so from the storm center. The air begins to spiral in toward the eyewall at point B. The storm center is an area of relatively low pressure, so the pressure on the air decreases as it travels from A to B. But it is always in contact with the sea surface, which acts as an almost unlimited heat reservoir, so its temperature is approximately constant. It is during this leg that enormous quantities of heat are added to the air. Leg A–B is the firebox of the hurricane.

At first it does not seem obvious that air is heated as it flows from A to B. But remember that air flowing from high to low pressure would cool were no heat added to it. Here, heat flowing from the sea into the air keeps it at nearly constant temperature. But there is a much more important source of heat: the enormous flow of energy that occurs when seawater evaporates into the inflowing air. Just as in a steam engine, the heat from the ocean is used mostly to evaporate water. Evaporation of water is a very efficient way to transfer heat from one body to another. That is why you feel cold when you are wet, especially if it is windy and/or dry: evaporation is taking heat from your body.

Thus the addition of heat in leg A–B shows up mostly as an increase in the humidity of the air. This form of heat is called *latent heat. Evaporation of seawater into the inflowing air is by far the most important source of heat driving the hurricane.* And while evaporation occurs over most of the area covered by the storm, the part of the evaporation that is actually effective in driving the storm occurs very near the hurricane's eyewall, where the winds are strongest. When hurricanes make landfall, they quickly die because they are cut off from their oceanic energy source.

There is one further effect on the energy of hurricanes. The terrific winds blowing across the surface near the eyewall are constantly being dissipated by friction. Just as you produce heat when you rub your hands together, this frictional dissipation heats the air near the surface. We shall return to this point in a moment.

The colors in Figure 10.2 show the entropy of the moist air; very loosely, this measures the total heat content of the air (including the latent component). Note that as air approaches the eyewall, its entropy increases rapidly, reflecting the large input of heat from the ocean.

Now at point B, the air turns and flows upward through the towering cumulonimbus clouds that make up the hurricane's eyewall. It is here that the latent heat is converted into sensible heat as water vapor condenses. But it is important to note that although there is enormous conversion between these two forms of heat, the *total* heat content (entropy, actually) remains approximately constant along this leg. (Note that the colors do not change along leg B–C.) As the air flows upward, the pressure on it decreases very rapidly. What we have here is an example of adiabatic expansion.

In the real world, the heat absorbed from the ocean and shot upwards through the hurricane's eyewall is expelled into the distant environment. But in computer models, we can place walls around the outside of the storm, forcing the air to return to the surface and thereby closing the loop. This makes it a little easier to describe the thermodynamic cycle of the storm.

By the time the air reaches point C in the high troposphere or lower stratosphere, some 12 to 18 km above the sea surface, the adiabatic expansion has lowered its temperature to a value close to that of the undisturbed upper tropical atmosphere, around -70°C or about 200 Kelvin. The air remains at approximately this temperature as it sinks down toward the tropopause (point D). Although the air is undergoing compression, it is losing heat by electromagnetic radiation to space (see Chapter 4). This leg of the cycle is very nearly one of isothermal compression.

Finally, the air sinks from point D back to the starting point A. In reality, the air is losing heat by radiation to space, just as in leg C–D, but it turns out that the amount of heat lost is almost equivalent to the amount of heat that would have been lost if the rainwater, instead of falling out of the storm, had remained in the air, evaporating as the air descends. This latter is, once again, just a conversion between two different forms of heat that preserves the total heat content. Thus leg D–A is very nearly one of adiabatic compression.

The thermodynamic cycle of a mature hurricane is almost exactly like the idealized cycle envisioned by Carnot. The hurricane would be a perfect steam engine but for one interesting feature.

In Carnot's cycle, as in real steam engines, the mechanical energy produced by the engine is used to do something outside the engine itself, like power a locomotive. If the locomotive is moving at constant speed, then there is an equilibrium between the mechanical energy production by the engine and the frictional dissipation of energy of the wheels on the tracks, the air moving past the train, and all the moving parts of the engine and drivetrain. This frictional dissipation turns the kinetic energy into heat, which is lost to the environment.

The mechanical energy produced by the hurricane's heat engine shows up as the energy of the winds. But in a mature hurricane, almost all of the frictional dissipation occurs in the inflow layer. Thus the power of the winds is converted back into heat, which then flows back into the system at the high temperature reservoir where heat energy is being injected in the first place. Thus, unlike in the locomotive, some of what would have been wasted heat energy is recycled back into the front end of the heat engine. This recycling of waste heat makes hurricanes somewhat more powerful than they would be otherwise.

We can use our understanding of the hurricane's Carnot cycle to estimate how strong the winds can get, assuming that the sea surface temperature remains unaffected by the storm. First, the rate at which kinetic energy is dissipated in the atmosphere near the surface is given by

$$D \approx C_D \rho V^3, \tag{10.2}$$

where D is the rate of energy dissipation per unit area of the surface, ρ is the density of the air, V is the wind speed and C_D is a number called the *drag coefficient*, which is a property of the surface itself and increases with the roughness of the surface. Thus, the dissipation of kinetic energy increases rapidly with wind speed. Now the rate at which heat is added to the inflowing air depends on two processes: evaporation from the ocean and the conversion of kinetic energy back into heat. The total rate of heat input per unit area of the earth's surface is given by

$$Q \approx C_K \rho V E + C_D \rho V^3, \tag{10.3}$$

where the new symbols are C_K, the *enthalpy exchange coefficient*, and E, the evaporative potential of the sea surface. The first is a number like the drag coefficient, but it measures how quickly heat flows across the air-sea interface. The second is a measure of the potential for transferring heat from the ocean to the atmosphere and is a function both of the air-sea temperature difference (which is usually a small effect in the Tropics) and the relative humidity of the air near the surface. The lower the relative humidity, the more water can evaporate into the air and the bigger the value of E. In fact, E is a measure of the air-sea thermodynamic disequilibrium and is a direct consequence of the greenhouse effect (see Chapter 4). The more greenhouse gases (and clouds) in the atmosphere, the less heat can escape from the ocean by means of radiation and the more heat has to get out by evaporation, thus the larger the value of E.

According to equation (10.1), the amount of work produced by the hurricane's Carnot cycle is just the heat input, given by equation (10.3), multiplied by the thermodynamic efficiency, $\left(T_{hot} - T_{cold}\right) / T_{hot}$:

$$W \approx (C_K \rho V E + C_D \rho V^3) \times \left(\frac{T_{hot} - T_{cold}}{T_{hot}}\right). \quad (10.4)$$

Equating the work given by equation (10.4) to the dissipation of kinetic energy given by equation (10.2) and doing a little algebra gives a formula for the maximum wind speed in a hurricane:

$$V_{max} \approx \sqrt{\left(\frac{T_{hot} - T_{cold}}{T_{cold}}\right) E}. \quad (10.5)$$

Although we have derived this equation by approximate arguments, the same equation has been derived exactly using a different approach published in the professional literature. Note that now T_{cold} rather than T_{hot} appears in the bottom part of the fraction in equation (10.5). This is a consequence of allowing the feedback effect of dissipative heating, and it makes V_{max} larger than it would otherwise be. We shall refer to the maximum wind given by equation (10.5) as the *potential intensity* of the hurricane. To calculate it, we need to know the ratio of the surface exchange coefficients, the modified thermodynamic efficiency, and E. To get this last number, all we need to know is the sea surface temperature, the temperature and humidity of the air just above the surface, and the surface pressure at the location of maximum winds. Unfortunately, we do not know in advance what this surface pressure will be; it depends on the maximum wind speed itself. The solution to this problem is to make a guess at the surface pressure, calculate the maximum wind speed from equation (10.5), use that to make a new estimate of the surface pressure at the location of maximum winds, use that to make a new estimate of E, plug that back into (10.5), and keep going around this loop. Eventually, the estimates of wind speed and surface pressure converge to the correct answer, unless the sea surface is extremely hot and/or T_{cold} is very small. In that case, there is no solution to the equation. What happens under these conditions is a runaway feedback: the lower the surface pressure, the more heat input results from the isothermal expansion of inflowing air; this additional heat input makes the storm more intense, dropping the surface pressure even lower, resulting in yet more heat input from isothermal expansion, and so on. The additional dissipation of kinetic energy simply cannot keep up. Although the conditions under which this would happen are far from those found on earth, we have simulated them with a computer model. The resulting storms, which we nicknamed *hypercanes*, collapse down to very tight vortices, with eyes only a few kilometers in diameter and with wind speeds approaching or even exceeding the speed of sound. These storms, rather than extending upward only into the very low stratosphere, would reach up to 30–40 km altitude, where they would deposit vast quantities of water in a layer of air that is normally very dry. This could cause a series of chemical reactions that would largely destroy the ozone layer that protects us from lethal ultraviolet radiation.

Besides E, we need to know the sea surface temperature, T_{hot}, and the temperature where air is flowing out of the top of the storm, T_{cold}. To estimate these, we need to know the vertical temperature profile of the atmosphere in the distant environment of the storm. Fortunately, this can be provided by weather balloons and other means.

Finally, we need to know the ratio C_K/C_D. We know that at very low wind speeds, this

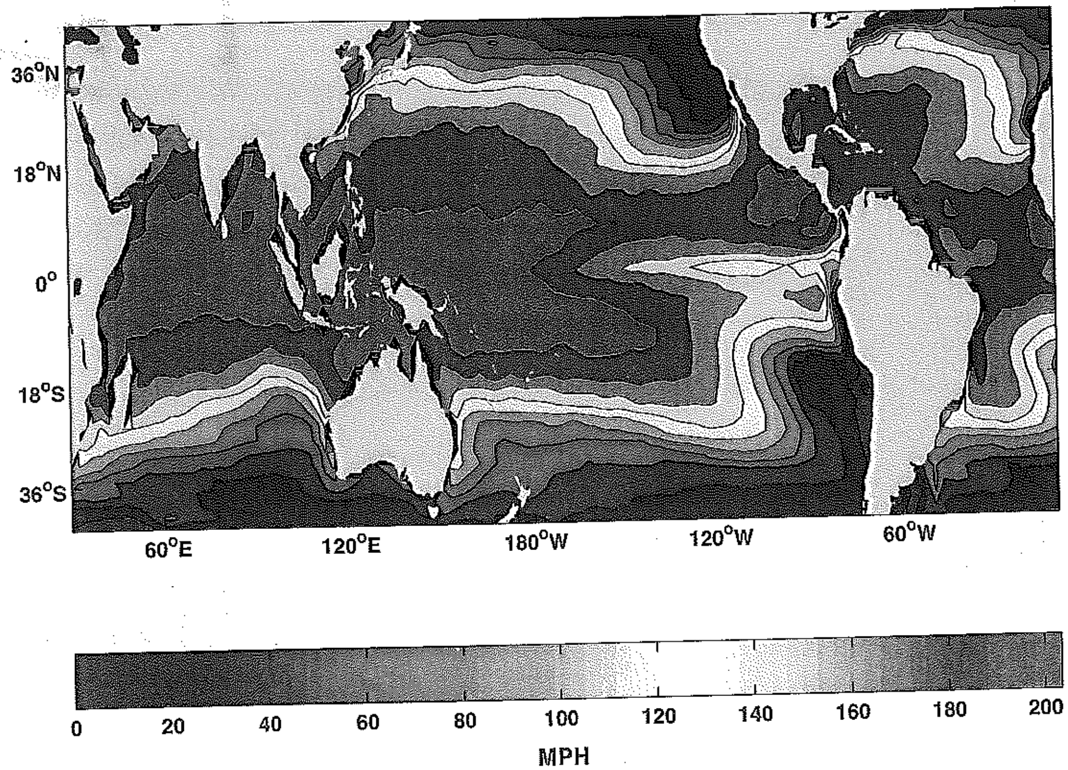


Figure 10.3: Map showing the maximum wind speed (in mph) achievable by hurricanes over the course of an average year, according to Carnot's theory of heat engines.

ratio is close to one. However, as the wind increases, waves develop on the ocean, making it rougher and increasing the drag coefficient, C_D. Up to wind speeds of around 60 mph, measurements show that this increase in C_D is enough to lower the ratio C_K/C_D to around 1/2. But using a value of 1/2 in equation (10.5) gives wind speeds that are too small to explain real hurricanes. Computer models show that at hurricane-force wind speeds, the ratio of exchange coefficients should be close to one, suggesting that some other processes are at work. Many research scientists believe that the missing ingredient is sea spray, which can transport enormous quantities of heat from the ocean to the atmosphere, as well as serving as an additional source of drag. The physics of air-sea interaction at hurricane wind speeds is an important research issue today.

Using climatological records of atmospheric temperature and humidity, and sea surface temperature, and assuming some value of C_K/C_D, we can estimate the maximum wind speed that can be achieved in hurricanes. Figure 10.3 shows the maximum wind speed that can occur over the course of a year, assuming monthly mean climatological conditions. Deep in the Tropics, hurricanes can apparently have winds as high as 200 mph. The potential intensity of storms decreases rapidly, poleward of around 30° latitude. Comparing this map to the map of genesis locations (see Chapter 14), it is clear that hurricanes only develop where the potential intensity is large.

The hurricane Carnot heat engine, with its recycling of waste heat, is one of the most efficient natural generators of power on earth. The amount of power dissipated by a typical mature Atlantic hurricane, as given by equation (10.2) integrated over the whole surface area covered by the storm, is on the order of 3 trillion Watts, which happens to be very nearly equal to the worldwide electrical generation capacity as of January 1996. This is enough to light 30 billion 100-Watt light bulbs. A Pacific supertyphoon can dissipate ten times this power.

As Carnot states at the opening of this chapter, the flow of heat drives almost everything that happens in the earth's atmosphere and oceans, and also deep within its interior. Among these phenomena, an intense hurricane comes closest to Carnot's ideal heat engine.

class 3: 1^{st} law, Carno 3.3 - 3.6 AIRS:

Bolton paper skewT

assign HW: 3.23
3.26
3.41
3.46 + 3.47

wyoming site - AIRS -

$$\Delta z = \frac{R_d \overline{T_v}}{g_0} \ln\left(\frac{P_1}{P_2}\right); \quad T_v = (1 + 0.61\, q)\, T$$

1^{st} thermo law:

$$\underbrace{du}_{\text{change in internal energy}} = \underbrace{dq}_{\text{heat}} - \underbrace{dW}_{\text{work}}$$

in meteorology, used to:

- understand redistribution of energy (e.g. latent vs sensible heat, ~~geopotential~~ buoyancy... lapse rate...
- understand the hydrological cycle, (Clausius Clapeyron eqn)
- heat engine (hurricanes) (carnot cycle)

$$du = dq - dW$$

dW — assumed to be entirely from a change in V ("pressure - volume work")

Work: units of energy: $= \int F \cdot ds = \int pA\,ds = \int p \cdot dV$ or $\int p\, d\alpha$ "per unit mass"

Carnot - 1824 weight × height

Joule - 1845 - used a paddle wheel to measure work + temp. increase of water, to come up w/ Joule's constant.

work - defined by external variables, can also include
- besides P-V - magnetic flux density + magnetization
mole fraction + chemical potential
(~~Cofthings~~ these macroscopic variables always come in conjugate pairs)

$du = dq - dw.$

dq: heat: microscopic energy transfers.

radiation, conduction, mixing, friction, viscosity, phase change
air poor conductor; surface sensible heat fluxes ~ few cm. turbulent frictional dissipation (latent heat release)

convection? not really - a bulk flow.

du: internal energy.

heat capacity: $\left.\frac{dq}{dT}\right|_{\text{volume constant}} \equiv c_v = \left.\frac{du}{dT}\right|_v$ 'cause $dw=0$

related to # of degrees of freedom - more complex molecules have higher c_v's.

$c_{v,\text{air}} = 717 \frac{J}{K \cdot kg}$

$35 \frac{J}{mol \cdot K}$ $c_{v,CH_4}: \sim 1250$

$c_p = \left.\frac{dq}{dT}\right|_{\text{pressure constant}}$:

$dq = du + dw = c_v dT + d(p\alpha) - \alpha dp$
$= c_v dT + R dT - \alpha dp$

$\rightarrow c_p = c_v + R$

1004 717 287

$p = \frac{F}{A} = \frac{mg}{A}.$

Special cases: $dq = du + dw$.

isobaric: $dp = 0$ $\quad dq = c_p dT = dh$ $\quad$ adiabatic: $dq = 0$.

enthalpy: $h \equiv u + p\alpha$

$\uparrow$ enthalpy, another state variable.

going back to: ~~dry static energy~~ $\cancel{dq = d(h+\phi) = d(c_pT + \phi)}$

$d(\cancel{h+\phi} \cancel{= dh + d\phi =} du$

$$dq = c_p dT - \underbrace{\alpha dp}_{d\Phi = g dz} = d(h + \Phi)$$

$h + \Phi =$ dry static energy, conserved when $dq = 0$.

can get the dry adiabatic lapse rate this way:

$$d(c_pT + \Phi) = 0$$

$$c_p dT + g dz = 0 \rightarrow \frac{dT}{dz} = -\frac{g}{c_p}$$

θ: air temperature of parcel at 1000 hPa.

$$c_p dT - \alpha dp = 0$$

substitute $\alpha = \frac{RT}{p}$ + divide by RT

$$\frac{c_p}{R} \frac{dT}{T} - \frac{dp}{p} = 0$$

integrate: $\displaystyle \frac{c_p}{R} \int_{T @ 1000\,hPa = \theta}^{T} \frac{dT}{T} = \int_{p_0}^{p} \frac{dp}{p} = \ln \frac{p}{p_0} = \frac{c_p}{R} \ln \frac{T}{\theta}$

p_0

$$\rightarrow \left(\frac{T}{\theta}\right)^{c_p/R} = \frac{p}{p_0} \rightarrow \theta = T\left(\frac{p_0}{p}\right)^{R/c_p} \qquad \frac{R}{c_p} \sim 0.286$$

$$= T\left(\frac{p_0}{p}\right)^{0.286}$$

θ conserved in a well mixed layer.

application: a subsiding air mass, ~~cooling radiatively by ~1 K/day.~~ moving down adiabatically.

moisture parameters:

(Petty

mixing ratio $w \equiv \frac{m_v}{m_d}$ 0-20 $\frac{g}{kg}$ on thermo charts

spec. humidity $q = \frac{m_v}{m_v + m_d} = \frac{w}{1+w}$

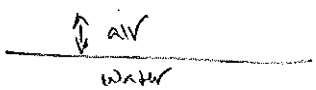
$T_v \doteq (1 + 0.61 q) T$ - Petty.
or $= (1 + 0.61 w) T$ in WH.

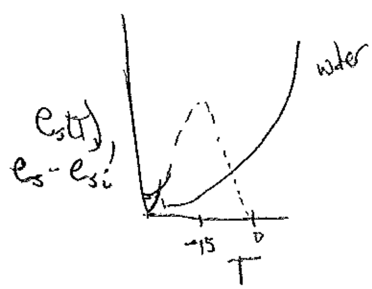
~~(Petty and WH use w + q interchangeably differently.~~

<u>saturation vapor pressure</u>

saturated w.r.t. a plane surface of water

$$e_s(T) = e_{s0} \exp\left[\frac{L}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$





Bolton & eqn (10)

$$e_s(T) = 6.112 \exp\left[\frac{17.67 T}{T + 243.5}\right]$$

mb or hPa

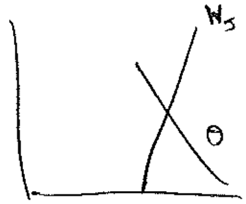
celsius

sat. mixing ratio

$$w_s \equiv \frac{m_{vs}}{m_d} \simeq 0.622 \frac{e_s}{p - e_s}$$

$$RH = 100 \frac{w}{w_s}$$

LCL.



Bolton.

ex. 3.50
saturated lapse rates

$$dq = du + dw$$
$$= c_p\, dT + g\, dz \quad = 0 \text{ if dry adiabatic}$$
$$= -L_v\, dw_s \text{ if saturated.}$$

$$\frac{dT}{dz} = \frac{-L_v}{c_p}\frac{dw_s}{dz} - \frac{g}{c_p}$$

$$= \frac{-L_v}{c_p\, dz}\left[\left.\frac{dw_s}{dp}\right|_T dp + \left.\frac{dw_s}{dT}\right|_p dT\right] - \frac{g}{c_p} \qquad ; \quad W_s = 0.622\,\frac{e_s}{p}$$

$$\left.\frac{dw_s}{dT}\right|_p > \left.\frac{dw_s}{dp}\right|_T$$

$$\Gamma_s = -\frac{dT}{dz} = \frac{g}{c_p}\left[\frac{1 - \rho L_v\left(\frac{dw_s}{dp}\right)_T}{1 + \frac{L}{c_p}\left(\frac{dw_s}{dT}\right)_p}\right]$$

$$\Gamma_s = \frac{g}{c_p}\left[\frac{1}{1 + \underbrace{\frac{L}{c_p}\left(\frac{dw_s}{dT}\right)_p}_{\text{reduced by this amount.}}}\right]$$

θ_e: eq. θ

$$-\frac{L_v\, dw_s}{T} = \frac{c_p\, dT}{T} + \frac{g\, dz}{T} = \frac{d\theta}{\theta} \approx -d\left(\frac{L_v w_s}{c_p T}\right)$$

$$\theta_e = \theta \exp\left(\frac{L_v w_s}{c_p T_{LCL}}\right)$$: θ of a parcel of air w/ all water vapor condensed out.

$$MSE = c_p T + \Phi + L_v q.$$

Bolton: eqn 21:

$$T_L = \frac{2840}{3.5 \ln T_k - \ln e - 4.805} + 55$$